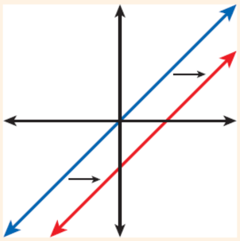
Chapter 1.3; Transforming Linear Functions

objective: the student will be able to solve problems involving linear transformations

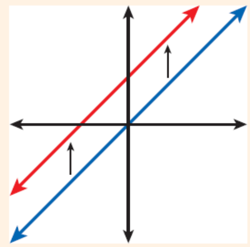
You have learned to transform functions by transforming each point.
Transformations can also be expressed by using function notation.

Horizontal Shift of $|h|$ Units



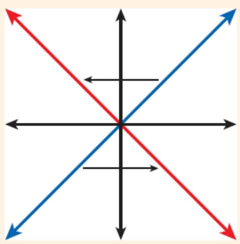
Input value changes.
$f(x) \rightarrow f(x - h)$
$h > 0$ moves right $h < 0$ moves left

Vertical Shift of $|k|$ Units



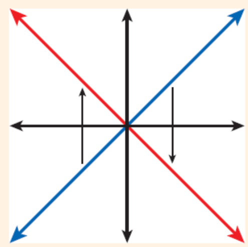
Output value changes.
$f(x) \rightarrow f(x) + k$
$k > 0$ moves up
$k < 0$ moves down

Reflection Across *y*-axis



Input value changes.
$f(x) \rightarrow f(-x)$
The lines are symmetric about the *y*-axis.

Reflection Across *x*-axis



Output value changes.
$f(x) \rightarrow -f(x)$
The lines are symmetric about the *x*-axis.

Example: Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

1. $f(x) = x - 2$, horizontal translation right 3 units

2. **$f(x) = 3x + 1$; translation 2 units right**

3.. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.
linear function defined in the table; reflection across the x-axis

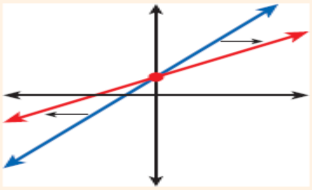
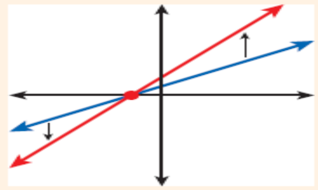
x	–2	0	2
$f(x)$	0	1	2

4. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.
linear function defined in the table; reflection across the x-axis

X	–1	0	1
$f(x)$	1	2	3

Stretches and compressions change the slope of a linear function. If the line becomes steeper, the function has been stretched vertically or compressed
horizontally. If the line becomes flatter, the function has been compressed vertically or stretched horizontally.

Stretches and Compressions

Horizontal	Vertical
Horizontal Stretch/Compression by a Factor of *b*	**Vertical Stretch/Compression by a Factor of *a***
 Input value changes. $f(x) \rightarrow f\left(\frac{1}{b}x\right)$	 Output value changes. $f(x) \rightarrow a \cdot f(x)$
$b > 1$ stretches away from the y-axis. $0 < \lvert b \rvert < 1$ compresses toward the y-axis.	$a > 1$ stretches away from the x-axis. $0 < \lvert a \rvert < 1$ compresses toward the x-axis.

These don't change!

- y–intercepts in a horizontal stretch or compression
- x–intercepts in a vertical stretch or compression

Example

5. Let $g(x)$ be a horizontal compression of $f(x) = -x + 4$ by a factor of . Write the rule for $g(x)$, and graph the function.

6. Let $g(x)$ be a vertical compression of $f(x) = 3x + 2$ by a factor of . Write the rule for $g(x)$ and graph the function.

Some linear functions involve more than one transformation by applying individual transformations one at a time in the order in which they are given.
For multiple transformations, create a temporary function—such as $h(x)$ in Example 7 below—to represent the first transformation, and then transform it to find the combined transformation.

Example:

7. **Let $g(x)$ be a horizontal shift of $f(x) = 3x$ left 6 units followed by a horizontal stretch by a factor of 4. Write the rule for $g(x)$.**