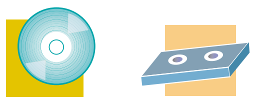
SECTION 2.4

Literal Equations and Their Applications

2.4 OBJECTIVES

1. *Solve a literal equation for any one of its variables*
2. *Solve applications involving geometric figures*
3. *Solve mixture problems*
4. *Solve motion problems*



Formulas are extremely useful tools in any field in which mathematics is applied. Formulas are simply equations that express a relationship between more than one letter or variable. You are no doubt familiar with all kinds of formulas, such as

$A = \frac{1}{2}bh$	The area of a triangle
$I = Prt$	Interest
$V = \pi r^2 h$	The volume of a cylinder

Actually a formula is also called a **literal equation** because it involves several letters or variables. For instance, our first formula or literal equation, $A = \frac{1}{2}bh$, involves the three letters A (for area), b (for base), and h (for height).

Unfortunately, formulas are not always given in the form needed to solve a particular problem. Then algebra is needed to change the formula to a more useful equivalent equation, which is solved for a particular letter or variable. The steps used in the process are very similar to those you used in solving linear equations. Let's consider an example.

Example 1

Solving a Literal Equation Involving a Triangle

Suppose that we know the area A and the base b of a triangle and want to find its height h.

We are given

$$A = \frac{1}{2}bh$$

Our job is to find an equivalent equation with h, the unknown, by itself on one side. We call $\frac{1}{2}b$ the **coefficient** of h. We can remove the two *factors* of that coefficient, $\frac{1}{2}$ and b, separately.

Note:

$$2\left(\frac{1}{2}bh\right) = \left(2 \cdot \frac{1}{2}\right)(bh) = 1 \cdot bh = bh$$

$$2A = 2\left(\frac{1}{2}bh\right)$$ Multiply both sides by 2 to clear the equation of fractions.

or

$$2A = bh$$

$$\frac{2A}{b} = \frac{bh}{b}$$ Divide by b to isolate h.

$$\frac{2A}{b} = h$$

or

$$h = \frac{2A}{b}$$ Reverse the sides to write h on the left.

We now have the height h in terms of the area A and the base b. This is called **solving the equation for h** and means that we are rewriting the formula as an equivalent equation of the form

$$h = \square$$

Here $\square$ means an expression containing all the numbers or letters *other than* h.

✓ CHECK YOURSELF 1

Solve $V = \frac{1}{3}Bh$ for h.

You have already learned the methods needed to solve most literal equations or formulas for some specified variable. As Example 1 illustrates, the rules of Sections 2.2 and 2.3 are applied in exactly the same way as they were applied to equations with one variable.

You may have to apply both the addition and the multiplication properties when solving a formula for a specified variable. Example 2 illustrates this situation.

Example 2

Solving a Literal Equation

This is a linear equation in two variables. You will see this again in Chapter 3.

Solve $y = mx + b$ for x.

Remember that we want to end up with x alone on one side of the equation. Let's start by subtracting b from both sides to undo the addition on the right.

$$y = mx + b$$
$$y - b = mx + b - b$$
$$y - b = mx$$

If we now divide both sides by m, then x with be alone on the right side.

$$\frac{y - b}{m} = \frac{mx}{m}$$
$$\frac{y - b}{m} = x$$

or

$$x = \frac{y - b}{m}$$

✓ *CHECK YOURSELF 2*

Solve $v = a + gt$ for t.

Let's summarize the steps illustrated by our examples.

Solving a Formula or Literal Equation

Step 1 If necessary, multiply both sides of the equation by the same term to clear the equation of fractions.

Step 2 Add or subtract the same term on both sides of the equation so that all terms involving the variable that you are solving for are on one side of the equation and all other terms are on the other side.

Step 3 Divide both sides of the equation by the coefficient of the variable that you are solving for.

Let's look at one more example, using the above steps.

Example 3

Solving a Literal Equation Involving Money

This is a formula for the *amount* of money in an account after interest has been earned.

Solve $A = P + Prt$ for r.

$$A = P + Prt$$

$$A - P = P - P + Prt$$

Subtracting P from both sides will leave the term involving r alone on the right.

$$A - P = Prt$$

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

Dividing both sides by Pt will isolate r on the right.

$$\frac{A - P}{Pt} = r$$

or

$$r = \frac{A - P}{Pt}$$

✓ *CHECK YOURSELF 3*

Solve $2x + 3y = 6$ for y.

Now let's look at an application of solving a literal equation.

Example 4

Solving a Literal Equation Involving Money

Suppose that the amount in an account, 3 years after a principal of \$5000 was invested, is \$6050. What was the interest rate?

From our previous example,

$$A = P + Prt \quad \textbf{(1)}$$

where A is the amount in the account, P is the principal, r is the interest rate, and t is the time in years that the money has been invested. By the result of Example 3 we have

$$r = \frac{A - P}{Pt} \quad \textbf{(2)}$$

Do you see the advantage of having our equation solved for the desired variable?

and we can substitute the known values in equation (2):

$$r = \frac{6050 - 5000}{(5000)(3)}$$

$$= \frac{1050}{15{,}000} = 0.07 = 7\%$$

The interest rate was 7%.

✓ *CHECK YOURSELF 4*

Suppose that the amount in an account, 4 years after a principal of \$3000 was invested, is \$3720. What was the interest rate?

In our subsequent applications, we will use the five-step process first described in Section 2.1. As a reminder, here are those steps.

To Solve Word Problems

Step 1 Read the problem carefully. Then reread it to decide what you are asked to find.

Step 2 Choose a letter to represent one of the unknowns in the problem. Then represent all other unknowns of the problem with expressions that use the same letter.

Step 3 Translate the problem to the language of algebra to form an equation.

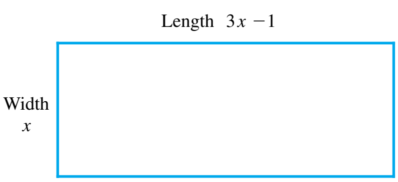
Step 4 Solve the equation and answer the question of the original problem.

Step 5 Check your solution by returning to the original problem.

Example 5

Solving a Geometry Application

Whenever you are working on an application involving geometric figures, you should draw a sketch of the problem, including the labels assigned in step 2.



The length of a rectangle is 1 centimeter (cm) less than 3 times the width. If the perimeter is 54 cm, find the dimensions of the rectangle.

Step 1 You want to find the dimensions (the width and length).

Step 2 Let x be the width.

Then $3x - 1$ is the length.

3 times the width ($3x$) — 1 less than (-1)

Step 3 To write an equation, we'll use this formula for the perimeter of a rectangle:

$$P = 2W + 2L \qquad \text{or} \qquad 2W + 2L = P$$

So

$$\underbrace{2x}_{\text{Twice the width}} + \underbrace{2(3x - 1)}_{\text{Twice the length}} = \underbrace{54}_{\text{Perimeter}}$$

Step 4 Solve the equation.

$$2x + 2(3x - 1) = 54$$
$$2x + 6x - 2 = 54$$
$$8x = 56$$
$$x = 7$$

Be sure to return to the original statement of the problem when checking your result.

The width x is 7 cm, and the length, $3x - 1$, is 20 cm. We leave step 5, the check, to you.

✓ *CHECK YOURSELF 5*

The length of a rectangle is 5 inches (in.) more than twice the width. If the perimeter of the rectangle is 76 in., what are the dimensions of the rectangle?

You will also often use parentheses in solving *mixture problems*. Mixture problems involve combining things that have a different value, rate, or strength. Look at Example 6.

Example 6

Solving a Mixture Problem

Four hundred tickets were sold for a school play. General admission tickets were \$4, while student tickets were \$3. If the total ticket sales were \$1350, how many of each type of ticket were sold?

Step 1 You want to find the number of each type of ticket sold.

Step 2 Let x be the number of general admission tickets.

We subtract x, the number of general admission tickets, from 400, the total number of tickets, to find the number of student tickets.

Then $\underbrace{400 - x}_{\text{400 tickets were sold in all.}}$ student tickets were sold.

Step 3 The sales value for each kind of ticket is found by multiplying the price of the ticket by the number sold.

General admission tickets: $4x$ — \$4 for each of the x tickets

Student tickets: $3(400 - x)$ — \$3 for each of the $400 - x$ tickets

So to form an equation, we have

$$\underbrace{4x}_{\text{Value of general admission tickets}} + \underbrace{3(400 - x)}_{\text{Value of student tickets}} = \underbrace{1350}_{\text{Total value}}$$

Step 4 Solve the equation.

$$4x + 3(400 - x) = 1350$$
$$4x + 1200 - 3x = 1350$$
$$x + 1200 = 1350$$
$$x = 150$$

This shows that 150 general admission and 250 student tickets were sold. We leave the check to you.

✓ *CHECK YOURSELF 6*

Beth bought 35¢ stamps and 15¢ stamps at the post office. If she purchased 60 stamps at a cost of \$17, how many of each kind did she buy?

The next group of applications we will look at in this section involves *motion problems*. They involve a distance traveled, a rate or speed, and time. To solve motion problems, we need a relationship among these three quantities.

Suppose you travel at a rate of 50 miles per hour (mi/h) on a highway for 6 hours (h). How far (what distance) will you have gone? To find the distance, you multiply:

Be careful to make your units consistent. If a rate is given in *miles per hour,* then the time must be given in *hours* and the distance in *miles.*

$$\underbrace{(50 \text{ mi/h})}_{\text{Speed or rate}}\underbrace{(6 \text{ h})}_{\text{Time}} = \underbrace{300 \text{ mi}}_{\text{Distance}}$$

In general, if r is the rate, t is the time, and d is the distance traveled, then

$$d = r \cdot t$$

This is the key relationship, and it will be used in all motion problems. Let's see how it is applied in Example 7.

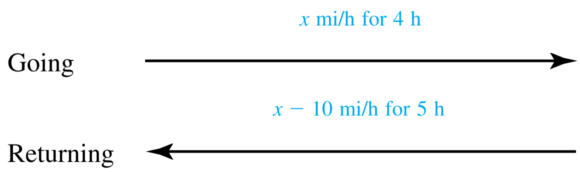
Example 7 Solving a Motion Problem

On Friday morning Ricardo drove from his house to the beach in 4 h. In coming back on Sunday afternoon, heavy traffic slowed his speed by 10 mi/h, and the trip took 5 h. What was his average speed (rate) in each direction?

Step 1 We want the speed or rate in each direction.

Step 2 Let x be Ricardo's speed to the beach. Then $x - 10$ is his return speed.

It is always a good idea to sketch the given information in a motion problem. Here we would have



Step 3 Since we know that the distance is the same each way, we can write an equation, using the fact that the product of the rate and the time each way must be the same.

So

$$\text{Distance (going)} = \text{distance (returning)}$$

$$\text{Time} \cdot \text{rate (going)} = \text{time} \cdot \text{rate (returning)}$$

$$\underbrace{4x}_{\substack{\text{Time} \cdot \text{rate} \\ \text{(going)}}} = \underbrace{5(x - 10)}_{\substack{\text{Time} \cdot \text{rate} \\ \text{(returning)}}}$$

A chart can help summarize the given information. We begin by filling in the information given in the problem.

	Rate	Time	Distance
Going	x	4	
Returning	$x - 10$	5	

Now we fill in the missing information. Here we use the fact that $d = rt$ to complete the chart.

	Rate	Time	Distance
Going	x	4	$4x$
Returning	$x - 10$	5	$5(x - 10)$

From here we set the two distances equal to each other and solve as before.

Step 4 Solve.

$$4x = 5(x - 10)$$
$$4x = 5x - 50$$
$$-x = -50$$
$$x = 50 \text{ mi/h}$$

x was his rate going; x − 10, his rate returning.

So Ricardo's rate going to the beach was 50 mi/h, and his rate returning was 40 mi/h.

Step 5 To check, you should verify that the product of the time and the rate is the same in each direction.

✓ *CHECK YOURSELF 7*

A plane made a flight (with the wind) between two towns in 2 h. Returning against the wind, the plane's speed was 60 mi/h slower, and the flight took 3 h. What was the plane's speed in each direction?

Example 8 illustrates another way of using the distance relationship.

Example 8

Solving a Motion Problem

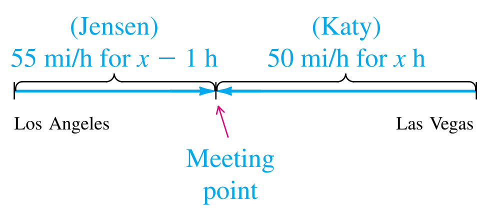
Katy leaves Las Vegas for Los Angeles at 10 A.M., driving at 50 mi/h. At 11 A.M. Jensen leaves Los Angeles for Las Vegas, driving at 55 mi/h along the same route. If the cities are 260 mi apart, at what time will they meet?

Step 1 Let's find the time that Katy travels until they meet.

Step 2 Let x be Katy's time.

Then $x - 1$ is Jensen's time. Jensen left 1 hr later!

Again, you should draw a sketch of the given information.



Step 3 To write an equation, we will again need the relationship $d = rt$. From this equation, we can write

$$\text{Katy's distance} = 50x$$

$$\text{Jensen's distance} = 55(x - 1)$$

As before, we can use a table to solve.

	Rate	Time	Distance
Katy	50	x	$50x$
Jensen	55	$x - 1$	$55(x - 1)$

From the original problem, the sum of those distances is 260 mi, so

$$50x + 55(x - 1) = 260$$

Step 4

$$50x + 55(x - 1) = 260$$
$$50x + 55x - 55 = 260$$
$$105x - 55 = 260$$
$$105x = 315$$
$$x = 3 \text{ h}$$

Be sure to answer the question asked in the problem.

Finally, since Katy left at 10 A.M., the two will meet at 1 P.M. We leave the check of this result to you.

✓ CHECK YOURSELF 8

At noon a jogger leaves one point, running at 8 mi/h. One hour later a bicyclist leaves the same point, traveling at 20 mi/h in the opposite direction. At what time will they be 36 mi apart?

✓ CHECK YOURSELF ANSWERS

1. $h = \dfrac{3V}{B}$. **2.** $t = \dfrac{v - a}{g}$.

3. $y = \dfrac{6 - 2x}{3}$ or $y = -\dfrac{2}{3}x + 2$.

4. The interest rate was 6%. **5.** The width is 11 in.; the length is 27 in. **6.** 40 at 35¢, and 20 at 15¢. **7.** 180 mi/h with the wind and 120 mi/h against the wind. **8.** At 2 P.M.

Exercises ▪ 2.4

Solve each literal equation for the indicated variable.

1. $P = 4s$ (for s) Perimeter of a square
2. $V = Bh$ (for B) Volume of a prism
3. $E = IR$ (for R) Voltage in an electric circuit
4. $I = Prt$ (for r) Simple interest
5. $V = LWH$ (for H) Volume of a rectangular solid
6. $V = \pi r^2 h$ (for h) Volume of a cylinder
7. $A + B + C = 180$ (for B) Measure of angles in a triangle
8. $P = I^2 R$ (for R) Power in an electric circuit
9. $ax + b = 0$ (for x) Linear equation in one variable
10. $y = mx + b$ (for m) Slope-intercept form for a line
11. $s = \frac{1}{2}gt^2$ (for g) Distance
12. $K = \frac{1}{2}mv^2$ (for m) Energy
13. $x + 5y = 15$ (for y) Linear equation
14. $2x + 3y = 6$ (for x) Linear equation
15. $P = 2L + 2W$ (for L) Perimeter of a rectangle
16. $ax + by = c$ (for y) Linear equation in two variables
17. $V = \frac{KT}{P}$ (for T) Volume of a gas
18. $V = \frac{1}{3}\pi r^2 h$ (for h) Volume of a cone
19. $x = \frac{a + b}{2}$ (for b) Average of two numbers
20. $D = \frac{C - s}{n}$ (for s) Depreciation
21. $F = \frac{9}{5}C + 32$ (for C) Celsius/Fahrenheit
22. $A = P + Prt$ (for t) Amount at simple interest
23. $S = 2\pi r^2 + 2\pi rh$ (for h) Total surface area of a cylinder
24. $A = \frac{1}{2}h(B + b)$ (for b) Area of a trapezoid

25. **Height of a solid.** A rectangular solid has a base with length 8 centimeters (cm) and width 5 cm. If the volume of the solid is 120 cm^3, find the height of the solid. (See Exercise 5.)

26. **Height of a cylinder.** A cylinder has a radius of 4 inches (in.). If the volume of the cylinder is 144 π $in.^3$, what is the height of the cylinder? (See Exercise 6.)

27. **Interest rate.** A principal of $3000 was invested in a savings account for 3 years. If the interest earned for the period was $450, what was the interest rate? (See Exercise 4.)

28. **Length of a rectangle.** If the perimeter of a rectangle is 60 feet (ft) and the width is 12 ft, find its length.

29. **Temperature conversion.** The high temperature in New York for a particular day was reported at 77°F. How would the same temperature have been given in degrees Celsius? (See Exercise 21.)

30. **Garden length.** Rose's garden is in the shape of a trapezoid. If the height of the trapezoid is 16 meters (m), one base is 20 m, and the area is 224 m^2, find the length of the other base. (See Exercise 24.)

Translate each of the following statements to equations. Let x represent the number in each case.

31. Twice the sum of a number and 4 is 20.

32. The sum of twice a number and 4 is 20.

33. 3 times the difference of a number and 5 is 21.

34. The difference of 3 times a number and 5 is 21.

35. The sum of twice an integer and 3 times the next consecutive integer is 48.

36. The sum of 4 times an odd integer and twice the next consecutive odd integer is 46.

Solve the following word problems.

37. **Number problem.** One number is 8 more than another. If the sum of the smaller number and twice the larger number is 46, find the two numbers.

38. **Number problem.** One number is 3 less than another. If 4 times the smaller number minus 3 times the larger number is 4, find the two numbers.

39. **Number problem.** One number is 7 less than another. If 4 times the smaller number plus 2 times the larger number is 62, find the two numbers.

40. Number problem. One number is 10 more than another. If the sum of twice the smaller number and 3 times the larger number is 55, find the two numbers.

41. Consecutive integers. Find two consecutive integers such that the sum of twice the first integer and 3 times the second integer is 28. (*Hint:* If x represents the first integer, $x + 1$ represents the next consecutive integer.)

42. Consecutive integers. Find two consecutive odd integers such that 3 times the first integer is 5 more than twice the second. (*Hint:* If x represents the first integer, $x + 2$ represents the next consecutive odd integer.)

43. Dimensions of a rectangle. The length of a rectangle is 1 inch (in.) more than twice its width. If the perimeter of the rectangle is 74 in., find the dimensions of the rectangle.

44. Dimensions of a rectangle. The length of a rectangle is 5 centimeters (cm) less than 3 times its width. If the perimeter of the rectangle is 46 cm, find the dimensions of the rectangle.

45. Garden size. The length of a rectangular garden is 4 meters (m) more than 3 times its width. The perimeter of the garden is 56 m. What are the dimensions of the garden?

46. Size of a playing field. The length of a rectangular playing field is 5 feet (ft) less than twice its width. If the perimeter of the playing field is 230 ft, find the length and width of the field.

47. Isosceles triangle. The base of an isosceles triangle is 3 cm less than the length of the equal sides. If the perimeter of the triangle is 36 cm, find the length of each of the sides.

48. Isosceles triangle. The length of one of the equal legs of an isosceles triangle is 3 in. less than twice the length of the base. If the perimeter is 29 in., find the length of each of the sides.

49. Ticket sales. Tickets for a play cost \$8 for the main floor and \$6 in the balcony. If the total receipts from 500 tickets were \$3600, how many of each type of ticket were sold?

50. Ticket sales. Tickets for a basketball tournament were \$6 for students and \$9 for nonstudents. Total sales were \$10,500, and 250 more student tickets were sold than nonstudent tickets. How many of each type of ticket were sold?

51. Number of stamps. Maria bought 80 stamps at the post office in 35¢ and 20¢ denominations. If she paid \$23.50 for the stamps, how many of each denomination did she buy?

52. Money denominations. A bank teller had a total of 125 \$10 bills and \$20 bills to start the day. If the value of the bills was \$1650, how many of each denomination did he have?

53. **Ticket sales.** Tickets for a train excursion were \$120 for a sleeping room, \$80 for a berth, and \$50 for a coach seat. The total ticket sales were \$8600. If there were 20 more berth tickets sold than sleeping room tickets and 3 times as many coach tickets as sleeping room tickets, how many of each type of ticket were sold?

54. **Baseball tickets.** Admission for a college baseball game is \$6 for box seats, \$5 for the grandstand, and \$3 for the bleachers. The total receipts for one evening were \$9000. There were 100 more grandstand tickets sold than box seat tickets. Twice as many bleacher tickets were sold as box seat tickets. How many tickets of each type were sold?

55. **Driving speed.** Patrick drove 3 hours (h) to attend a meeting. On the return trip, his speed was 10 miles per hour (mi/h) less and the trip took 4 h. What was his speed each way?

56. **Bicycle speed.** A bicyclist rode into the country for 5 h. In returning, her speed was 5 mi/h faster and the trip took 4 h. What was her speed each way?

57. **Driving speed.** A car leaves a city and goes north at a rate of 50 mi/h at 2 P.M. One hour later a second car leaves, traveling south at a rate of 40 mi/h. At what time will the two cars be 320 mi apart?

58. **Bus distance.** A bus leave a station at 1 P.M., traveling west at an average rate of 44 mi/h. One hour later a second bus leaves the same station, traveling east at a rate of 48 mi/h. At what time will the two buses be 274 mi apart?

59. **Traveling time.** At 8:00 A.M., Catherine leaves on a trip at 45 mi/h. One hour later, Max decides to join her and leaves along the same route, traveling at 54 mi/h. When will Max catch up with Catherine?

60. **Bicycling time.** Martina leaves home at 9 A.M., bicycling at a rate of 24 mi/h. Two hours later, John leaves, driving at the rate of 48 mi/h. At what time will John catch up with Martina?

61. **Traveling time.** Mika leaves Boston for Baltimore at 10:00 A.M., traveling at 45 mi/h. One hour later, Hiroko leaves Baltimore for Boston on the same route, traveling at 50 mi/h. If the two cities are 425 mi apart, when will Mika and Hiroko meet?

62. **Traveling time.** A train leaves town A for town B, traveling at 35 mi/h. At the same time, a second train leaves town B for town A at 45 mi/h. If the two towns are 320 mi apart, how long will it take for the two trains to meet?

63. **Tree inventory.** There are 500 Douglas fir and hemlock trees in a section of forest bought by Hoodoo Logging Co. The company paid an average of \$250 for each Douglas fir and \$300 for each hemlock. If the company paid \$132,000 for the trees, how many of each kind did the company buy?

64. **Tree inventory.** There are 850 Douglas fir and ponderosa pine trees in a section of forest bought by Sawz Logging Co. The company paid an average of \$300 for each Douglas fir and \$225 for each ponderosa pine. If the company paid \$217,500 for the trees, how many of each kind did the company buy?

65. There is a universally agreed on "order of operations" used to simplify expressions. Explain how the order of operations is used in solving equations. Be sure to use complete sentences.

66. A common mistake when solving equations is the following:

The equation: $2(x - 2) = x + 3$
First step in solving: $2x - 2 = x + 3$

Write a clear explanation of what error has been made. What could be done to avoid this error?

67. Another very common mistake is in the equation below:

The equation: $6x - (x + 3) = 5 + 2x$
First step in solving: $6x - x + 3 = 5 + 2x$
Write a clear explanation of what error has been made and what could be done to avoid the mistake.

68. Write an algebraic equation for the English statement "Subtract 5 from the sum of x and 7 times 3 and the result is 20." Compare you equation with other students. Did you all write the same equation? Are all the equations correct even though they don't look alike? Do all the equations have the same solution? What is wrong? The English statement is *ambiguous.* Write another English statement that leads correctly to more than one algebraic equation. Exchange with another student and see if they think the statement is ambiguous. Notice that the algebra is *not* ambiguous!