5-6 Graphing Inequalities in Two Variables

Then

You graphed linear equations. (Lesson 3-1)

Now

- Graph linear inequalities on the coordinate plane.
- Solve inequalities by graphing.

New Vocabulary

boundary
half-plane
closed half-plane
open half-plane

Math Online

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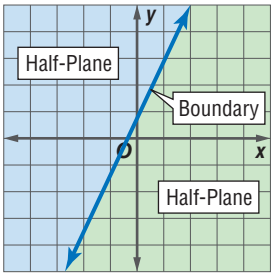
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

Hannah has budgeted $35 every three months for car maintenance. From this she must buy oil costing $3 and filters that cost $7 each. How much oil and how many filters can Hannah buy and stay within her budget?

Graph Linear Inequalities The graph of a linear inequality is the set of points that represent all of the possible solutions of that inequality. An equation defines a **boundary**, which divides the coordinate plane into two **half-planes**.



The boundary may or may not be included in the graph of an inequality. When it is included, the solution is a **closed half-plane**. When not included, the solution is an **open half-plane**.

Key Concept — Graphing Linear Inequalities

For Your FOLDABLE

Step 1 Graph the boundary. Use a solid line when the inequality contains $\leq$ or $\geq$. Use a dashed line when the inequality contains $<$ or $>$.

Step 2 Use a test point to determine which half-plane should be shaded.

Step 3 Shade the half-plane that contains the solution.

EXAMPLE 1 Graph an Inequality ($<$ or $>$)

Graph $3x - y < 2$.

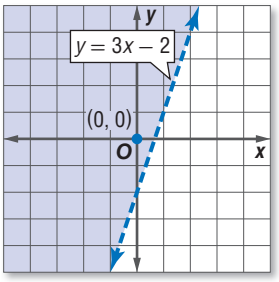
Step 1 First, solve for y in terms of x.

$$\begin{aligned} 3x - y &< 2 \\ -y &< -3x + 2 \\ y &> 3x - 2 \end{aligned}$$

Then, graph $y = 3x - 2$. Because the inequality involves $>$, graph the boundary with a dashed line.

Step 2 Select a test point in either half-plane. A simple choice is (0, 0).

$3x - y < 2$	Original inequality
$3(0) - 0 < 2$	$x = 0$ and $y = 0$
$0 < 2$	true



Step 3 So, the half-plane containing the origin is the solution. Shade this half-plane.

Check Your Progress Graph each inequality.

1A. $y > \frac{1}{2}x + 3$

1B. $x - 1 > y$

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StudyTip

Selecting a Test Point When selecting a test point, a standard choice is the origin because it offers easy calculations. However, if it lies on the border, you must choose another point that is not on the border.

EXAMPLE 2 Graph an Inequality ($\leq$ or $\geq$)

Graph $x + 5y \leq 10$.

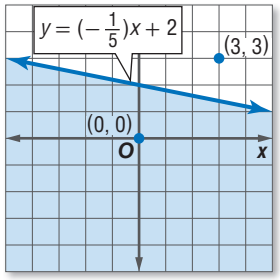
Step 1 Solve for y in terms of x.

$x + 5y \leq 10$	**Original inequality**
$5y \leq -x + 10$	**Subtract x from each side and simplify.**
$y \leq -\frac{1}{5}x + 2$	**Divide each side by 5.**

Graph $y = -\frac{1}{5}x + 2$. Because the inequality symbol is $\leq$, graph the boundary with a solid line.

Step 2 Select a test point. Let's use (3, 3). Substitute the values into the original inequality.

$x + 5y \leq 10$	**Original inequality**
$3 + 5(3) \leq 10$	**$x = 3$ and $y = 3$**
$18 \nleq 10$	**Simplify.**



Step 3 Since this statement is false, shade the other half-plane.

Check Your Progress **Graph each inequality.**

2A. $x - y \leq 3$ **2B.** $2x + 3y \geq 18$

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Solve Linear Inequalities We can use a coordinate plane to solve inequalities with one variable.

EXAMPLE 3 Solve Inequalities From Graphs

Use a graph to solve $3x + 5 < 14$.

Step 1 First graph the boundary, which is the related equation. Replace the inequality sign with an equals sign, and solve for x.

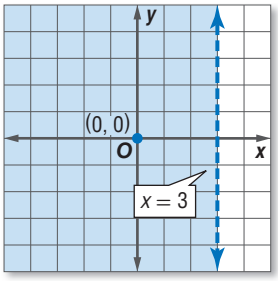
$3x + 5 < 14$	**Original inequality**
$3x + 5 = 14$	**Change $<$ to $=$.**
$3x = 9$	**Subtract 5 from each side and simplify.**
$x = 3$	**Divide each side by 3.**

Graph $x = 3$ with a dashed line.

Step 2 Choose (0, 0) as a test point. These values in the original inequality give us $5 < 14$.

Step 3 Since this statement is true, shade the half-plane that contains the point (0, 0).

Notice that the x-intercept of the graph is at 3. Since the half-plane to the left of the x-intercept is shaded, the solution is $x < 3$.



Check Your Progress **Use a graph to solve each inequality.**

3A. $4x - 3 \geq 17$ **3B.** $-2x + 6 > 12$

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When using inequalities to solve real-world problems, the domain and the range are often restricted to nonnegative or whole numbers.

Real-World EXAMPLE 4 Write and Solve an Inequality

CLASS PICNIC **A yearbook company promises to give the junior class a picnic if they spend at least $28,000 on yearbooks and class rings. Each yearbook costs $35, and each class ring costs $140. How many yearbooks and class rings must the junior class buy to get their picnic?**

Real-World Link

As a supplement to traditional yearbooks, many schools are producing digital versions. They include features that allow you to click on a picture and see a short video clip.

Source: *eSchool News*

Understand You know the cost of each item and the minimum amount the class needs to spend.

Plan Let x = the number of yearbooks and y = the number of class rings the class must buy. Write an inequality.

$35	times	the number of yearbooks	plus	$140	times	the number of rings	is at least	$28,000.
35	$\cdot$	x	$+$	140	$\cdot$	y	$\geq$	28,000

Solve Solve for y in terms of x.

$35x + 140y - 35x \geq 28{,}000 - 35x$ **Subtract 35x from each side.**

$140y \geq -35x + 28{,}000$ **Divide each side by 140.**

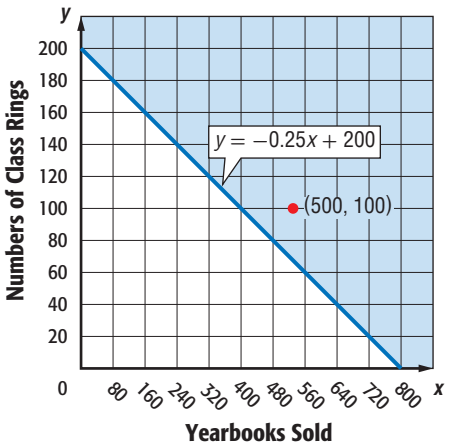
$\frac{140y}{140} \geq \frac{-35x}{140} + \frac{28000}{140}$ **Simplify.**

$y \geq -0.25x + 200$ **Simplify.**

Because the yearbook company cannot sell a negative number of items, the domain and range must be nonnegative numbers. Graph the boundary with a solid line. If we test (0, 0), the result is $0 \geq 28{,}000$, which is false. Shade the closed half-plane that does not include the origin. One solution is (500, 100), or 500 yearbooks and 100 class rings.

Problem-Solving Tip

Use a Graph You can use a graph to visualize data, analyze trends, and make predictions.



Check If we test (500, 100), the result is $100 \geq 75$, which is true. Because the company cannot sell a fraction of an item, only points with whole-number coordinates can be solutions.

Check Your Progress

4. **MARATHONS** Neil wants to run a marathon at a pace of at least 6 miles per hour. Write and graph an inequality for the miles y he will run in x hours.

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Check Your Understanding

Examples 1 and 2 pp. 315–316

Graph each inequality.

1. $y > x + 3$
2. $y \geq -8$
3. $x + y > 1$
4. $y \leq x - 6$
5. $y < 2x - 4$
6. $x - y \leq 4$

Example 3 p. 316

Use a graph to solve each inequality.

7. $7x + 1 < 15$
8. $-3x - 2 \geq 11$
9. $3y - 5 \leq 34$
10. $4y - 21 > 1$

Example 4 p. 317

11. **FINANCIAL LITERACY** The surf shop has a weekly overhead of $2300.
    a. Write an inequality to describe this situation.
    b. How many skimboards and longboards must the shop sell each week to make a profit?

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 2 pp. 315–316

Graph each inequality.

12. $y < x - 3$
13. $y > x + 12$
14. $y \geq 3x - 1$
15. $y \leq -4x + 12$
16. $6x + 3y > 12$
17. $2x + 2y < 18$
18. $5x + y > 10$
19. $2x + y < -3$
20. $-2x + y \geq -4$
21. $8x + y \leq 6$
22. $10x + 2y \leq 14$
23. $-24x + 8y \geq -48$

Example 3 p. 316

Use a graph to solve each inequality.

24. $10x - 8 < 22$
25. $20x - 5 > 35$
26. $4y - 77 \geq 23$
27. $5y + 8 \leq 33$
28. $35x + 25 < 6$
29. $14x - 12 > -31$

Example 4 p. 317

30. **DECORATING** Sybrina is decorating her bedroom. She has $300 to spend on paint and bed linens. A gallon of paint costs $14, while a set of bed linens costs $60.
    a. Write an inequality for this situation.
    b. How many gallons of paint and bed linen sets can Sybrina buy and stay within her budget?

Use a graph to solve each inequality.

31. $3x + 2 < 0$
32. $4x - 1 > 3$
33. $-6x - 8 \geq -4$
34. $-5x + 1 < 3$
35. $-7x + 13 < 10$
36. $-4x - 4 \leq -6$

37. **SOCCER** The girls' soccer team wants to raise $2000 to buy new goals. How many of each item must they sell to buy the goals?
    a. Write an inequality that represents this situation.
    b. Graph this inequality.
    c. Make a table of values that shows at least five possible solutions.
    d. Plot the solutions from part **c**.

Graph each inequality. Determine which of the ordered pairs are part of the solution set for each inequality.

38. $y \geq 6$; $\{(0, 4), (-2, 7), (4, 8), (-4, -8), (1, 6)\}$

39 $x < -4$; $\{(2, 1), (-3, 0), (0, -3), (-5, -5), (-4, 2)\}$

40. $2x - 3y \leq 1$; $\{(2, 3), (3, 1), (0, 0), (0, -1), (5, 3)\}$

41. $5x + 7y \geq 10$; $\{(-2, -2), (1, -1), (1, 1), (2, 5), (6, 0)\}$

42. $-3x + 5y < 10$; $\{(3, -1), (1, 1), (0, 8), (-2, 0), (0, 2)\}$

43. $2x - 2y \geq 4$; $\{(0, 0), (0, 7), (7, 5), (5, 3), (2, -5)\}$

44. **RECYCLING** A curbside recycling service will remove up to 50 pounds of plastic bottles and paper products each week. They charge \$0.25 per pound of plastic and \$0.75 per pound for paper products.

a. Write an inequality that describes the pounds of each kind of product that can be included in the curbside service.

b. Write an inequality that describes the charge.

c. Graph each inequality.

d. Compare the two graphs.

Real-World Link

The energy saved when recycling one glass bottle is enough to light a traditional light bulb for four hours.

Source: PlanetPals

45. **MULTIPLE REPRESENTATIONS** Use inequalities A and B to investigate graphing compound inequalities on a coordinate plane.

A. $7(y + 6) \leq 21x + 14$ **B.** $-3y \leq 3x - 12$

a. **NUMERICAL** Solve each inequality for y.

b. **GRAPHICAL** Graph both inequalities on one graph. Shade the half-plane that makes A true in red. Shade the half-plane that makes B true in blue.

c. **VERBAL** What does the overlapping region represent?

H.O.T. Problems

Use Higher-Order Thinking Skills

46. **FIND THE ERROR** Reiko and Kristin are solving $4y \leq \frac{8}{3}x$ by graphing. Is either of them correct? Explain your reasoning.

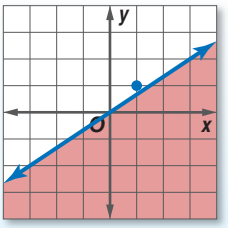
Reiko

$4y \leq \frac{8}{3}x$

$y \leq \frac{2}{3}x$

test point (1, 1)

$1 \leq \frac{2}{3}$ false



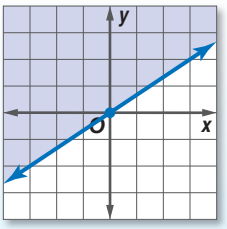
Kristin

$4y \leq \frac{8}{3}x$

$y \leq \frac{2}{3}x$

test point (0, 0)

$0 \leq 0$ true



47. **CHALLENGE** Graph $y > |x + 5|$.

48. **REASONING** Explain why a point on the boundary should not be used as a test point.

49. **OPEN ENDED** Write a two-variable inequality with a restricted domain and range to represent a real-world situation. Give the domain and range, and explain why they are restricted.

50. **WRITING IN MATH** Summarize the steps to graph an inequality in two variables.

Standardized Test Practice

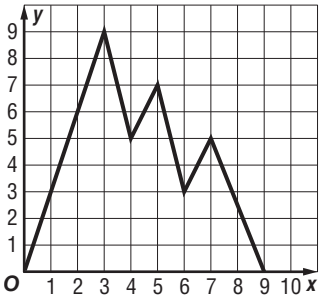
51. What is the domain of this function?

A $\{x \mid 0 \le x \le 3\}$

B $\{x \mid 0 \le x \le 9\}$

C $\{y \mid 0 \le y \le 9\}$

D $\{y \mid 0 \le y \le 3\}$



52. EXTENDED RESPONSE An arboretum will close for the winter when all of the trees have lost their leaves. The table shows the number of trees each day that still have leaves.

Day	5	10	15	20
Trees with Leaves	325	260	195	130

a. Write an equation that represents the number of trees with leaves y after d days.

b. Find the y-intercept. What does it mean in the context of this problem?

c. After how many days will the arboretum close? Explain how you got your answer.

53. Which inequality best represents the statement below?

A jar contains 832 gumballs. Ebony's guess was within 46 pieces.

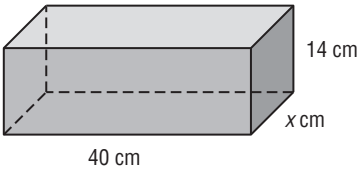
F $|g - 832| \le 46$

G $|g + 832| \le 46$

H $|g - 832| \ge 46$

J $|g + 832| \ge 46$

54. GEOMETRY If the rectangular prism has a volume of 10,080 cm^3, what is the value of x?



A 12

B 14

C 16

D 18

Spiral Review

Solve each open sentence. (Lesson 5-5)

55. $|y - 2| > 4$

56. $|t - 6| \le 5$

57. $|3 + d| < -4$

Solve each compound inequality. (Lesson 5-4)

58. $4c - 4 < 8c - 16 < 6c - 6$

59. $5 < \frac{1}{2}p + 3 < 8$

60. $0.5n \ge -7$ or $2.5n + 2 \le 9$

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

61. (1, −3) and (2, 5)

62. (−2, −4) and (−7, 3)

63. (−6, −8) and (−8, −5)

64. FITNESS The table shows the maximum heart rate to maintain during aerobic activities. Write an equation in function notation for the relation. Determine what would be the maximum heart rate to maintain in aerobic training for an 80-year-old. (Lesson 3-5)

Age (yr)	20	30	40	50	60	70
Pulse rate (beats/min)	175	166	157	148	139	130

Skills Review

65. WORK The formula $s = \frac{w - 10r}{m}$ is used to find keyboarding speeds. In the formula, s represents the speed in words per minute, w the number of words typed, r the number of errors, and m the number of minutes typed. Solve for r. (Lesson 2-8)